Quadratic Residue Diffuser

While working through the design of an Auditorium, we took the opportunity to study the types of acoustic surfaces typically used to tune and temper the walls and ceilings of performance spaces. In addition to the overall geometry of the room, the properties of these surfaces play an important role in how sound bounces around and within a space. In general, these surfaces are categorized based on how sound interacts with them and fall under the three general headings -- reflective, absorptive and diffusive. Reflective and absorptive elements fulfill definitive metrics, and are similarly specifically designed. Reflective elements are generally designed as hard flat surfaces and are intended to bounce a sound with minimal affect to its clarity and energy. Often made from softer materials, absorptive elements are very much the opposite and aim to capture any sound that collides with it. If these two are understood as extremes of an interaction spectrum, diffusive elements land somewhere within. Answering the question of where exactly a diffusive element lands presents interesting performative design opportunities. Diffusion is intended to contribute to the enjoyment of a critical listening environment by disrupting the clarity of a sound while maintaining a sound's energy. Its effect is a sense of spaciousness. In earlier auditoriums and theaters, diffusion occurred unsystematically through the use of ornament and decoration. As these spaces began to adopt clean lines and remove ornament, other architectural elements needed to be developed to take up the diffusive function.

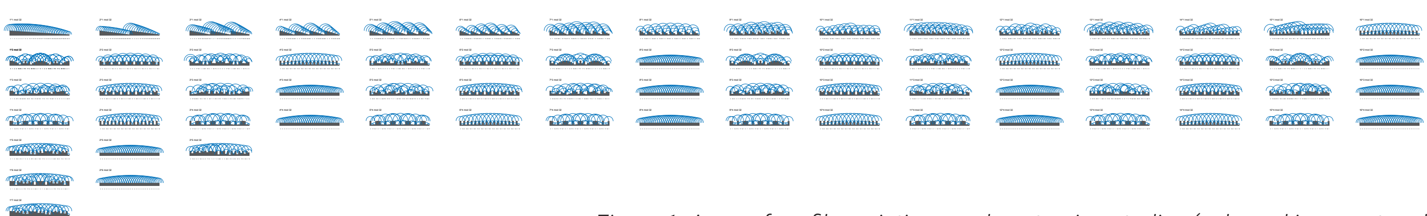


Figure 1. Array of profile variations and ray-tracing studies (enlarged image at end)

Figure 2. Ray-tracing studies (enlarged image at end)

After some research and discussions with consulting acousticians, we came upon the diffuser designs of physicist Manfred Schroeder. Schroeder's diffuser design is considered by some to be the most significant event in diffuser development. While most diffusion came as the result of textures, ornament and general architectural irregularities -- all of which made predictions about success difficult to be sure -- Schroeder used number sequences to dictate the relative dimensions of articulations and verified how sound interacts. His invention is referred to as a Quadratic Residue Diffuser, or QRD.

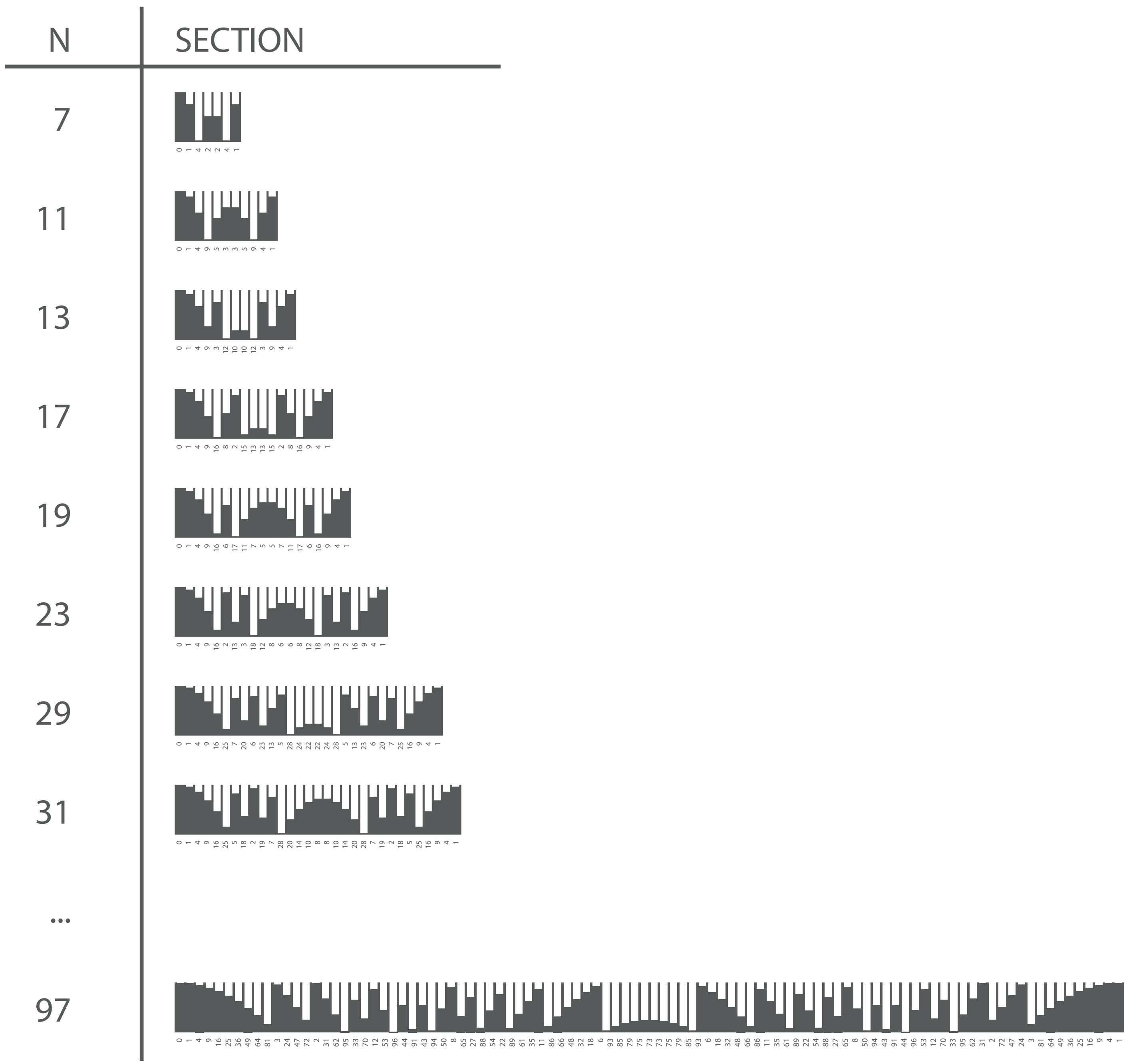


Figure 3. Sections of well depths as N increases

Unlike earlier diffusive elements, Schroeder's QRD made sound diffusion not only predictable, but also optimized. The underlying mathematics makes it possible to design a diffuser for a specific range of sound frequencies. The number sequence dictating the panel design results in sound reflections scattered neatly across a wide angle and minimizes the chance of unintended focal points. Because the wells are made with varying depths, sound will reflect off the individual bottoms at slightly different times. So in addition to being scattered spatially, sound is also broken up temporarily.

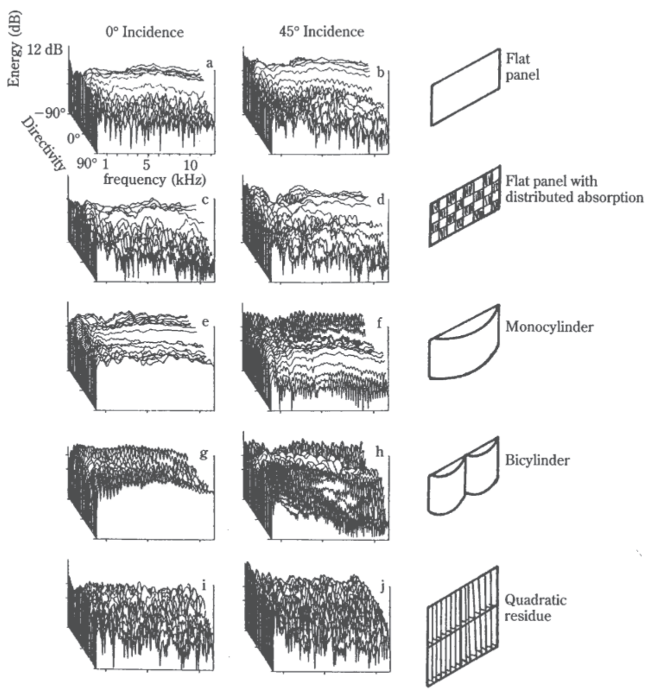


Figure 4. Comparison of diffusion profiles

$$\lambda = c / f$$

c: 343.2 m/s ; 1126 ft/s

f: frequency

$$\top \; f = c / (2 * \text{width})$$

$$\bot \; f = c / (4 * \text{depth})$$

$$z = x^2 \bmod N$$

$$z = (x^2 * y^2) \bmod N$$

Figure 5. Design equations for Schroeder's diffuser

Figure 5 contains the equations Schroeder used to design the Quadratic Residue Diffuser. N represents the number of wells in the QRD design. Any number can be used for x and y to generate a quadratic number sequence, but QRD designs simply use the number 1. QRD designs take into account the range of sound frequencies intended to be affected using frequency bounding equations to determine well dimensions. Well widths are dictated by the higher limit of sound frequencies to affect; the higher the frequency limit, the narrower the well widths. Meanwhile, well depths are determined by the lower frequency limit; the lower the frequency limit, the deeper the well.

Since its invention, QRD design has become standardized. Despite its parametric foundation, it is similarly constructed by both commercial producers and DIY communities. In part, this is due to manufacturing limitations. As a result, QRD designs commonly use smaller variables for N, with 7 being by far the most common. Larger numbers will increase the amount of unique well depths and the complexity of construction.

In most cases, when this diffuser is used in a space, identical panels are repeated along the length of the surface needing to be treated. Unfortunately, because of the repetition, the resulting diffused sound may reflect in such a way that overlapping occurs at some frequencies and along some angles, creating areas of noticeably higher and lower sound levels. Increasing the distance between repeating elements can help with the matter. An aperiodic organization of the diffusers may also work, but a single period device is preferred. Altogether, the issues that arise as a result of repetition can be avoided by increasing the N-value in the quadratic equation. In doing this, we can generate a symmetrical number sequence of any length needed. Using this method, we can stretch a single period over whatever surface we would like to mitigate and imbue a space with an ideal diffusion profile.

Given that well widths are dictated by the upper limit of the frequency range we'd like to affect, we can use that number to divide the length of the area we'd like to fill out with diffusive treatments. This number, rounded to the nearest prime number, is used as the N-value of the quadratic design equation. After generating the proper quadratic number sequence, the largest number should be associated to the deepest well, whose actual dimension is set using the frequency bounding equations. All other numbers in between will dictate correlating well depths as a fractional function.

To reiterate, one reason this approach has not been done before has been the prohibitive cost of manufacturing the multiplicities required. But given the recent advances of machine manufacturing, the limitation could now be considered negligible. It is typically far more economical to make one hundred copies of one thing than to make one hundred unique versions. With the use of computer automated fabrication techniques, the additional cost of doing the latter has been diminished considerably. Even though the effect of a diffusive element is typically only noticed by the most trained of ears in the most critical of listening environments, proper diffusion deployment should no longer be the result of fabrication limitations.

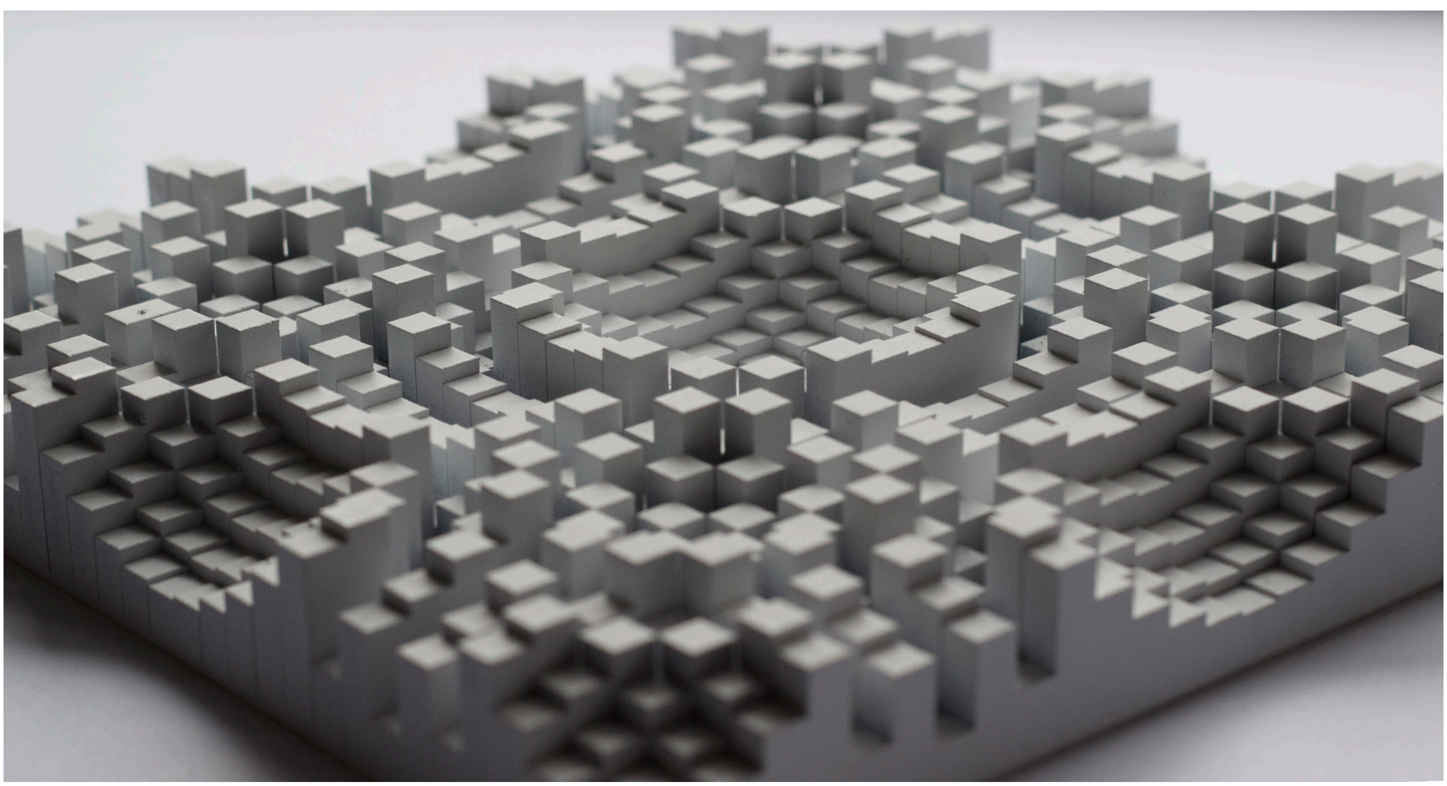


Figure 6. Scaled prototype

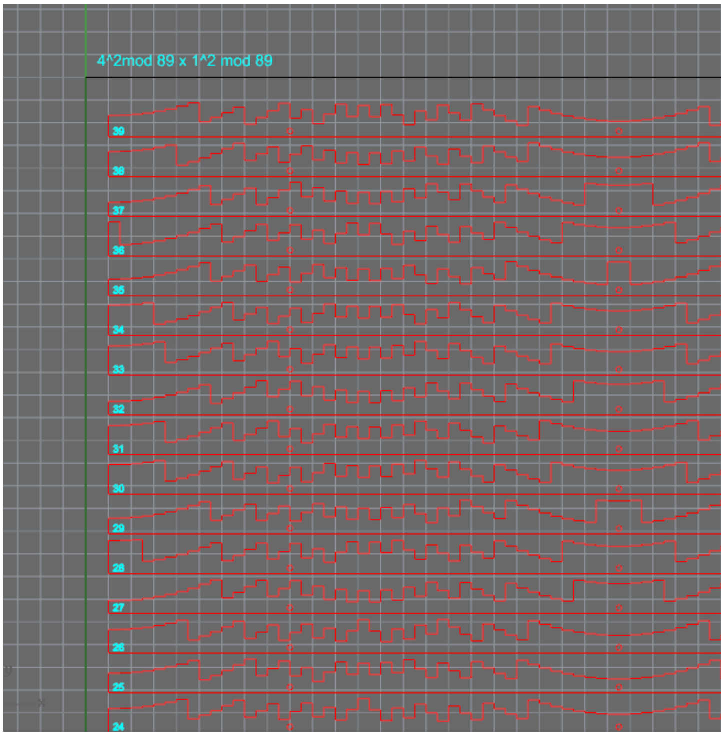
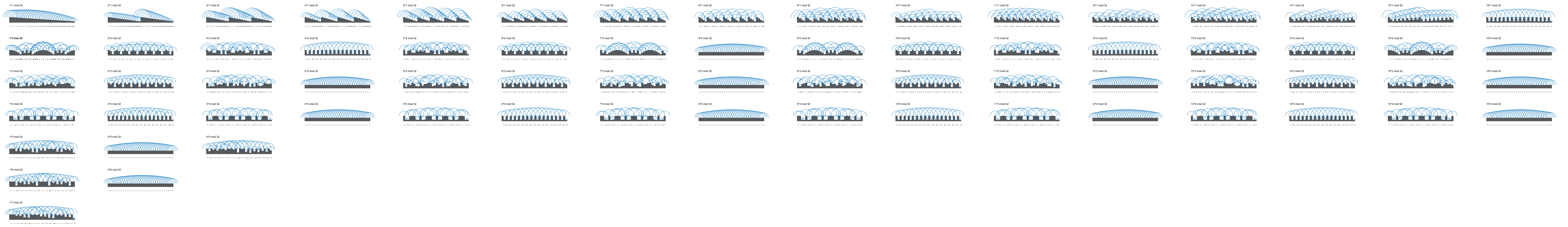


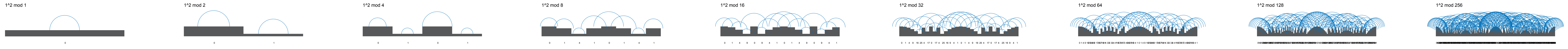
Figure 7. Screen capture

Figure 8. Scaled prototype

The fabrication method used to produce the prototypes could easily be adapted to produce the actual panels. After 3-dimensionally modeling the panels using custom algorithms based on the quadratic residue and frequency bounding equations, we extracted each unique profile along one direction of the panel. These profiles were then cut from some sheet material with a thickness equal to the desired well width. After stacking all the profiles together in their proper order, the panel is completed.

The initial intent of this research was to explore possibilities within acoustic design using a dynamic algorithmic modeling approach. Our focus narrowed on diffusion and the design of corresponding architectural elements. Narrowing even more, we developed a particular interest in Quadratic Residue Diffuser design and began extending its underlying mathematics. The process and understanding described above has been limited to this research effort. Beyond some rudimentary ray-tracing tests, we have yet to validate the effectiveness of any prototype generated. Though this research started with a focus on acoustic performance and followed into diffusion specifically, without any means to test our assumptions and invention, we pivoted to a more unconstrained exploration into the relationship between modulus formulas and patterning. In this way, divested of performance concerns, we applied the general techniques gained to a slightly more frenetic effort generating patterns and evaluating them from an aesthetic stand point.




1^2 mod 1
1^2 mod 2
1^2 mod 4
1^2 mod 8
1^2 mod 16
1^2 mod 32
1^2 mod 64
1^2 mod 128
1^2 mod 256